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Towards Minimized RMSE in Signal Calculation: Employing Correction Terms in Fourier Series

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Abstract

- **We add correction terms into the traditional Fourier series expansion to optimize signal processing and analysis.**
- The root-mean-square error (RMSE) as a function of frequency of target signal in low-frequency ranges (0 to 40 Hz) was investigate using one to three correction terms.

Enhanced Fourier Series by Correction Term

- The Fourier series expansion can be described as F(x) which includes g(x) below

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{N_1}\left(a_n cos\frac{2\pi nx}{T} + b_n sin\frac{2\pi nx}{T}\right) + g(x) \qquad (1)$$

F(x) : target function to be expanded
N_1 : the number of terms in Fourier expansion
T : the Fourier expansion period.

- Correction Term $g(x)$ here was represented as

$$g(x) = A_1(cos2\pi f_{c_1}x) + B_1(sin2\pi f_{c_1}x) + A_2(cos2\pi f_{c_2}x) + B_2(sin2\pi f_{c_2}x) + \cdots + A_{N_2}\left(cos2\pi f_{c_{N_2}}x\right) + B_{N_2}\left(sin2\pi f_{c_{N_2}}x\right) \qquad (2)$$

Selection of f_{c1}, f_{c2},…, f_{cN2}

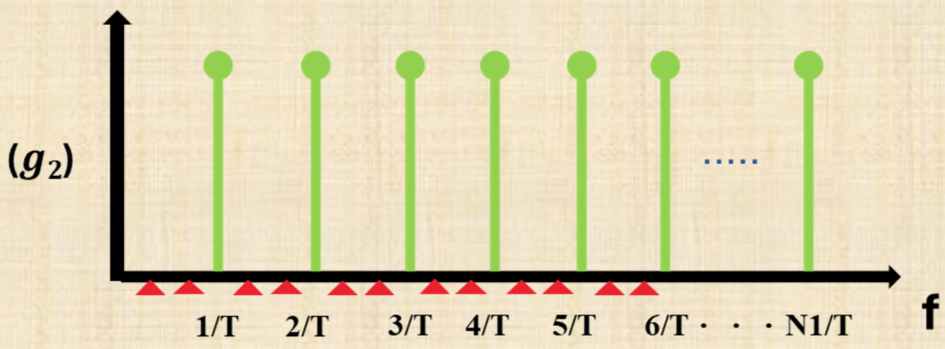


Fig. 1 The schematic of the Fourier series expansion adding the correction term basis.

- The frequencies of correction terms are situated between one expansion period of Fourier series.
- By this irregular frequency components, the signal function were expected to completely expressed in a limited bandwidth.

Coefficient obtained by Least Square Method

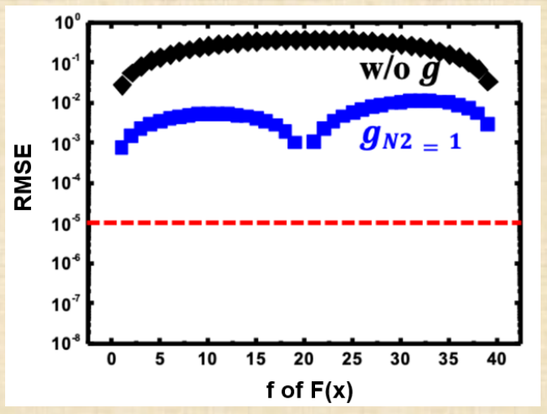
- The coefficients in both the Fourier series expansion and correction terms were obtained using Least Square Method and by partially differentiating the function of J below.

$$J\{(a_1, b_1 \dots a_{N1}, b_{N1}) \dots, (A_1, B_1 \dots A_{N1}, B_{N1})\}$$
$$= \int_{-T/2}^{T/2}\left[F(x) - \frac{a_0}{2} - \sum_{n=1}^{N1}(a_n cos\frac{2\pi nx}{T} + b_n sin\frac{2\pi nx}{T} - g(x)\right]^2 dx \qquad (3)$$

Relationship of RMSE and Frequency of Target Signal

- Adding one set of correction terms

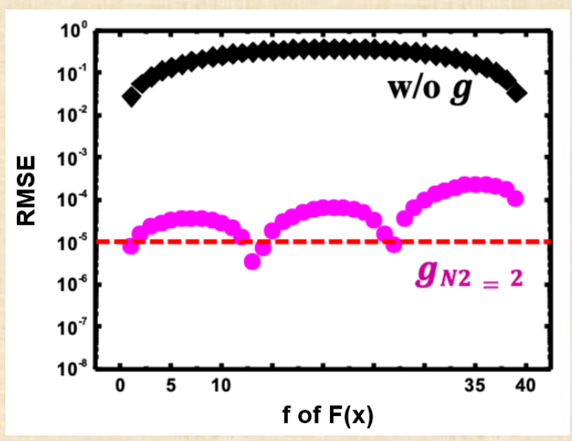
$$g(x) = A_1(cos2\pi 20x) + B_1(sin2\pi 20x)$$



- **Target function**
  $F(x) = \sqrt{0.5} \times cos(2\pi fx) + \sqrt{0.5} \times sin(2\pi fx)$
  - the f of the target function = 29.5 Hz.
  - The Fourier expansion period T = 1 sec
  - expansion over the range x = - 0.5 to 0.5.
  - cutoff frequency = 40 Hz.
- **N1= 1**

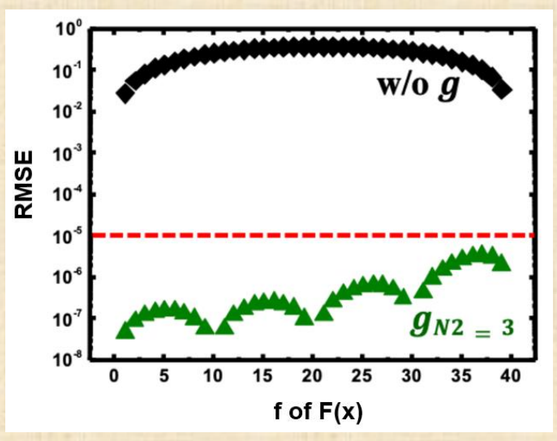
- two sets of correction terms

$$g(x) = A_1(cos2\pi 13.33x) + B_1(sin2\pi 13.33x) + A_2(cos2\pi 26.66x) + B_2(sin2\pi 26.66x)$$



- three sets of correction terms

$$g(x) = A_1(cos2\pi 10x) + B_1(sin2\pi 10x) + A_2(cos2\pi 20x) + B_2(sin2\pi 20x) + A_3(cos2\pi 30x) + B_3(sin2\pi 30x)$$



- **The RMSE can be reduced to below 10^{-5} by adding three correction terms in all low-frequency ranges from 0 Hz to 40 Hz.**
- **Improving RMSE at low-frequency ranges can help signal analysis for ECG data.**

References

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