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Enhancing Signal Processing with Correction Terms: A Novel Approach to Fourier Series Expansion

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Abstract

- **An innovative correction term basis that enhances the traditional Fourier series expansion was proposed.**
- This algorithm aims to reduce the root-mean-square error (RMSE) between the target and simulated function and improve signal processing and analysis.

Enhanced Fourier Series by Correction Term

- The enhanced Fourier series expansion can be represented as F(x)

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{N_1}\left(a_n cos\frac{2\pi nx}{T} + b_n sin\frac{2\pi nx}{T}\right) + g(x) \qquad (1)$$

F(x) : target function to be expanded

N_1 : the number of terms in Fourier expansion

T : the Fourier expansion period.

- Correction Term $g(x)$ was introduced and written as

$$g(x) = A_1\left(cos2\pi f_{c_1}x\right) + B_1\left(sin2\pi f_{c_1}x\right) + A_2\left(cos2\pi f_{c_2}x\right) + B_2\left(sin2\pi f_{c_2}x\right) + \cdots + A_{N_2}\left(cos2\pi f_{c_{N_2}}x\right) + B_{N_2}\left(sin2\pi f_{c_{N_2}}x\right) \qquad (2)$$

Selection of $f_{c1}, f_{c2}, \ldots, f_{cN2}$

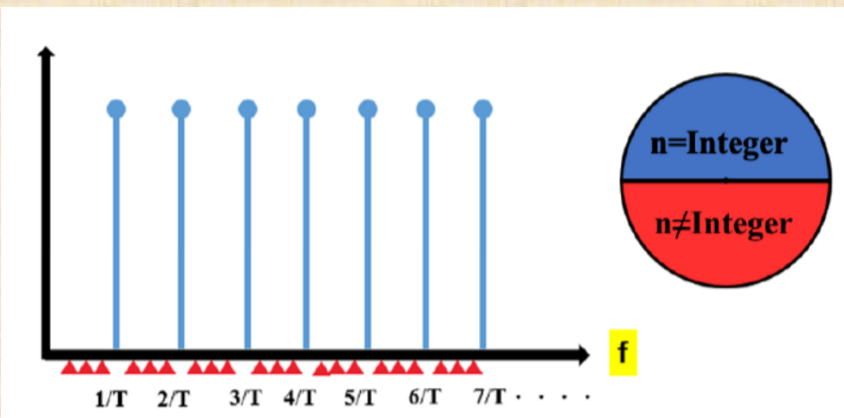


Fig. 1 The schematic of the Fourier series expansion adding the correction term basis.

- The frequencies of correction terms are located between an expansion period of Fourier series.
- By this irregular components, the signal function were expected to completely expressed within a limited bandwidth.

Coefficient obtained by Least Square Method

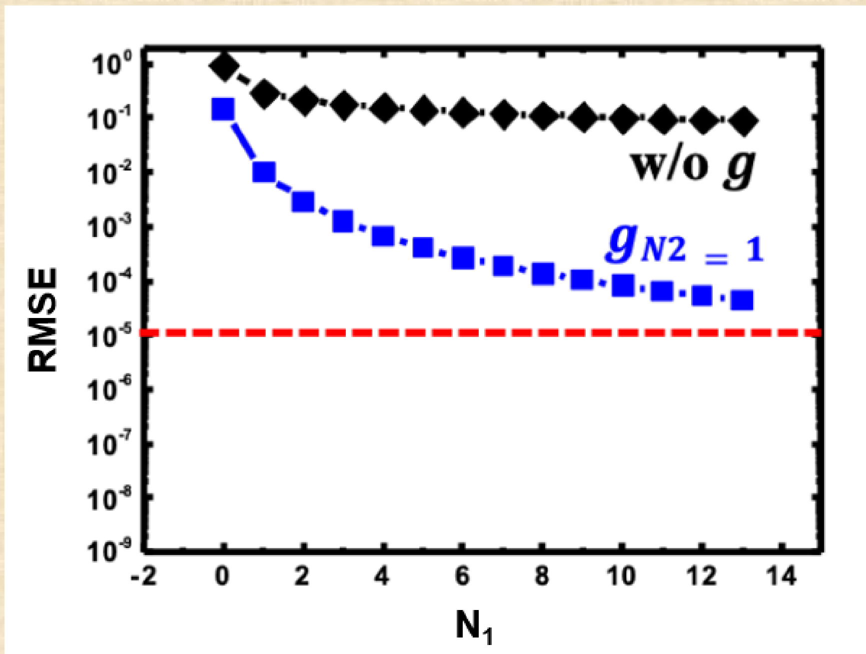
- The coefficients in both the Fourier series expansion and correction terms were obtained using Least Square Method and by partially differentiating the function of J below.

$$J\{(a_1, b_1 \ldots a_{N1}, b_{N1}) \ldots, (A_1, B_1 \ldots A_{N1}, B_{N1})\}$$
$$= \int_{-T/2}^{T/2}\left[F(x) - \frac{a_0}{2} - \sum_{n=1}^{N1}(a_n cos\frac{2\pi nx}{T} + b_n sin\frac{2\pi nx}{T} - g(x)\right]^2 dx \qquad (3)$$

RMSE of Simulated Results

- Adding one set of correction terms

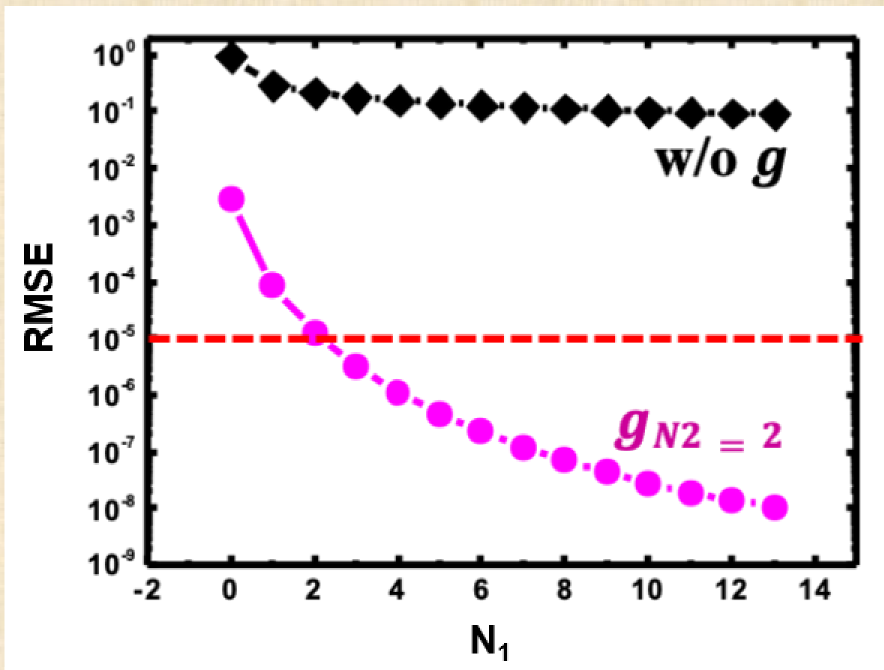
$$g(x) = A_1(cos2\pi 20x) + B_1(sin2\pi 20x)$$



- $F(x) = \sqrt{0.5} \times cos(2\pi f x) + \sqrt{0.5} \times sin(2\pi f x)$
- the f of the target function = 29.5 Hz.
- The Fourier expansion period $T = 1$ sec
- expansion over the range x = - 0.5 to 0.5.
- cutoff frequency = 40 Hz.

- Adding two sets of correction terms

$$g(x) = A_1(cos2\pi 13.33x) + B_1(sin2\pi 13.33x) + A_2(cos2\pi 26.66x) + B_2(sin2\pi 26.66x)$$



- **The RMSE can be reduced to below 10^{-5} by adding two correction terms.**
- **Fewer Fourier series expansion terms are required.**

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